

Consistent chiral kinetic theory in Weyl materials: chiral magnetic plasmons

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(Dated: October 26, 2016)

We argue that the correct definition of the electric current in the chiral kinetic theory for Weyl materials should include the Chern–Simons contribution that makes the theory consistent with the local conservation of the electric charge in electromagnetic and strain-induced pseudoelectromagnetic fields. By making use of such a kinetic theory, we study the plasma frequencies of collective modes in Weyl materials in constant magnetic and pseudomagnetic fields taking into account the effects of dynamical electromagnetism. We show that the collective modes are chiral plasmons. While the plasma frequency of the longitudinal collective mode coincides with the Langmuir one, this mode is unusual because it is characterized not only by oscillations of the electric current density, but also oscillations of the chiral current density. The latter are triggered by a dynamical version of the chiral electric separation effect. We also find that the plasma frequencies of the transverse modes split up in a magnetic field. This finding suggests an efficient means of extracting the chiral shift parameter from the measurement of the plasma frequencies in Weyl materials.

PACS numbers: 71.45.-d, 03.65.Sq

Introduction.— The study of the fundamental properties of magnetized relativistic matter attracted a lot of attention in recent years. The physical systems in question include the plasmas in the early Universe [1] and relativistic heavy-ion collisions [2, 3], degenerate states of dense matter in compact stars [4], and a growing number of recently discovered three-dimensional Dirac and Weyl materials [5–7]. To large extent, the recent increased activity in the studies of magnetized relativistic matter is driven by the hope of detecting macroscopic implications of quantum anomalies. One of such implications is the celebrated chiral magnetic effect (CME) [8], which has been detected indirectly in quark-gluon plasma created in heavy-ion collisions (for a review, see Ref. [3]), as well as in Dirac semimetals [9]. Note that the interpretation of the heavy-ion experiments is not without a controversy [10].

The search for macroscopic implications of quantum anomalies is greatly facilitated by the recent discovery of Dirac and Weyl materials, whose low-energy quasiparticle excitations are described by relativistic-like equations. Indeed, unlike most forms of truly relativistic matter, these novel condensed matter materials open the possibility for revealing and testing many anomalous effects in magnetized matter in table-top experiments under controlled conditions. Moreover, they may even allow for modeling phenomena that are impossible in relativistic physics. A specific example is provided by a background pseudomagnetic (or equivalently axial magnetic) field \mathbf{B}_5 that can be effectively produced by a mechanical strain in Dirac and Weyl materials [11–13]. In essence, the pseudomagnetic field \mathbf{B}_5 resembles the ordinary mag-

netic field \mathbf{B} , but acts on opposite chirality quasiparticles so as if they had opposite charges. In the case of the Dirac semimetal Cd_3As_2 , for example, the strength of the strain-induced pseudomagnetic field could range from about $B_5 \approx 0.3$ T in twisted nanowires [13] to $B_5 \approx 15$ T in bended thin films [14].

Collective excitations are simple, but informative probes of plasma properties [15]. It is natural to ask, therefore, whether such modes in chiral plasmas can be affected by quantum anomalies. The authors of Ref. [16] proposed that the chiral anomaly implies the existence of a new type of collective excitation, i.e., the chiral magnetic wave (CMW), that originates from an interplay of the chiral and electric charge density waves. In this study, we will investigate the collective modes in Weyl materials, using the framework of the chiral kinetic theory and a proper treatment of dynamical electromagnetism.

The central idea of this Letter is to use the correct definition of the electric current in the chiral kinetic theory for Weyl materials with strain-induced pseudoelectromagnetic fields. As we show, the current should necessarily include the Chern–Simons contribution, which is also known as the Bardeen–Zumino polynomial [17]. Such a correction restores the local conservation of the electric charge in the case of general electromagnetic and pseudoelectromagnetic fields. In addition, this topological term affects the properties of collective modes. For example, their plasma frequencies acquire a dependence on the chiral shift parameter, i.e., the momentum-space separation between the Weyl nodes.

Model.— The chiral kinetic theory is a semiclassical theory, which describes the time evolution of the one-

particle distribution functions f_λ for the right- ($\lambda = +$) and left-handed ($\lambda = -$) fermions. In the collisionless limit (assuming that the frequency of collective excitations ω is much larger than inverse relaxation time $1/\tau$), the kinetic equations for the chiral fermions are given by [18, 19]

$$\frac{\partial f_\lambda}{\partial t} + \frac{\left[e\tilde{\mathbf{E}}_\lambda + \frac{e}{c}(\mathbf{v} \times \mathbf{B}_\lambda) + \frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda \cdot \mathbf{B}_\lambda)\mathbf{\Omega}_\lambda \right] \cdot \nabla_{\mathbf{p}} f_\lambda}{1 + \frac{e}{c}(\mathbf{B}_\lambda \cdot \mathbf{\Omega}_\lambda)} + \frac{\left[\mathbf{v} + e(\tilde{\mathbf{E}}_\lambda \times \mathbf{\Omega}_\lambda) + \frac{e}{c}(\mathbf{v} \cdot \mathbf{\Omega}_\lambda)\mathbf{B}_\lambda \right] \cdot \nabla_{\mathbf{r}} f_\lambda}{1 + \frac{e}{c}(\mathbf{B}_\lambda \cdot \mathbf{\Omega}_\lambda)} = 0, \quad (1)$$

where $\mathbf{E}_\lambda = \mathbf{E} + \lambda\mathbf{E}_5$ and $\mathbf{B}_\lambda = \mathbf{B} + \lambda\mathbf{B}_5$ are effective electric and magnetic fields for fermions of chirality λ , $\mathbf{\Omega}_\lambda = \lambda\hbar\mathbf{p}/(2p^3)$ is the Berry curvature [20], $p \equiv |\mathbf{p}|$, $\tilde{\mathbf{E}}_\lambda = \mathbf{E}_\lambda - (1/e)\nabla_{\mathbf{r}}\epsilon_{\mathbf{p}}$, and the factor $1/[1 + e(\mathbf{B}_\lambda \cdot \mathbf{\Omega}_\lambda)/c]$ accounts for the correct phase-space density of chiral states in a (pseudo-)magnetic field [21]. By making use of the fermion dispersion relation, valid up to the linear order in the background magnetic field \mathbf{B}_λ ,

$$\epsilon_{\mathbf{p}} = v_F p \left[1 - \frac{e}{c}(\mathbf{B}_\lambda \cdot \mathbf{\Omega}_\lambda) \right], \quad (2)$$

we derive the quasiparticle velocity $\mathbf{v} = \nabla_{\mathbf{p}}\epsilon_{\mathbf{p}}$, i.e.,

$$\mathbf{v} = v_F \frac{\mathbf{p}}{p} \left[1 + 2\frac{e}{c}(\mathbf{B}_\lambda \cdot \mathbf{\Omega}_\lambda) \right] - \frac{ev_F}{cp} \mathbf{B}_\lambda (\mathbf{p} \cdot \mathbf{\Omega}_\lambda). \quad (3)$$

Here v_F is the Fermi velocity. From a physical viewpoint, pseudoelectromagnetic fields \mathbf{E}_5 and \mathbf{B}_5 can be induced by a strain in Dirac and Weyl materials [11–14]. A pseudoelectric field \mathbf{E}_5 , for example, can be generated by time-dependent deformations of a sample. A uniform pseudomagnetic field \mathbf{B}_5 can be produced by either applying a static torsion to a wire [13] or bending a thin film [14].

The equilibrium distribution functions for chiral fermions are given by the standard Fermi-Dirac distributions

$$f_\lambda^{(\text{eq})} = \frac{1}{e^{(\epsilon_{\mathbf{p}} - \mu_\lambda)/T} + 1}, \quad (4)$$

where T is the temperature (measured in energy units) and $\mu_\lambda = \mu + \lambda\mu_5$ are the effective chemical potentials for the right- and left-handed fermions. Note that μ and μ_5 are the electric and chiral chemical potentials, respectively. The distribution functions for antiparticles $\bar{f}_\lambda^{(\text{eq})}$ are obtained by replacing $\mu_\lambda \rightarrow -\mu_\lambda$. In addition, for antiparticles, one should replace $\mathbf{\Omega}_\lambda \rightarrow -\mathbf{\Omega}_\lambda$.

The charge and current densities are given by [19]

$$\rho_\lambda = e \int \frac{d^3p}{(2\pi\hbar)^3} \left[1 + \frac{e}{c}(\mathbf{B}_\lambda \cdot \mathbf{\Omega}_\lambda) \right] f_\lambda, \quad (5)$$

$$\mathbf{j}_\lambda = e \int \frac{d^3p}{(2\pi\hbar)^3} \left[\mathbf{v} + \frac{e}{c}(\mathbf{v} \cdot \mathbf{\Omega}_\lambda)\mathbf{B}_\lambda + e(\tilde{\mathbf{E}}_\lambda \times \mathbf{\Omega}_\lambda) \right] f_\lambda + e\nabla \times \int \frac{d^3p}{(2\pi\hbar)^3} f_\lambda \epsilon_{\mathbf{p}} \mathbf{\Omega}_\lambda, \quad (6)$$

where the last term in Eq. (6) describes a magnetization current.

Local charge (non-)conservation.— By using Eqs. (1), (5), and (6) together with the Maxwell's equations, one can easily derive the following continuity equations for the chiral and electric currents:

$$\frac{\partial \rho_5}{\partial t} + \nabla \cdot \mathbf{j}_5 = \frac{e^3}{2\pi^2\hbar^2 c} [(\mathbf{E} \cdot \mathbf{B}) + (\mathbf{E}_5 \cdot \mathbf{B}_5)], \quad (7)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c} [(\mathbf{E} \cdot \mathbf{B}_5) + (\mathbf{E}_5 \cdot \mathbf{B})]. \quad (8)$$

The first equation is related to the celebrated chiral anomaly [22] and expresses the nonconservation of the chiral charge in the presence of electromagnetic or pseudoelectromagnetic fields. Physically, this nonconservation can be understood as pumping of the chiral charge between the Weyl nodes of opposite chiralities. The second equation describes the anomalous local nonconservation of the *electric* charge in electromagnetic and pseudoelectromagnetic fields.

The local nonconservation of the electric charge is a very serious problem. If taken at face value, it would imply that the electric charge is literally created out of nothing. It was suggested in Refs. [12, 13] that it may correspond to pumping of the charge between the bulk and the boundary of the system. However, it is unclear how such a spatially nonlocal process could resolve the problem.

As we argue below, the resolution of the problem is much simpler. It lies in the fact that Eqs. (7) and (8) are the so-called *covariant* anomaly relations that come from the fermionic sector of the theory, in which left- and right-handed fermions are treated in a symmetric way. Just like in quantum field theory, this is inconsistent with the gauge symmetry. The correct physical currents, satisfying the local conservation of the electric charge, are the *consistent* currents [23]. A very clear discussion of these concepts in the framework of a low-energy effective theory is given in Ref. [24]. Clearly, the same should apply to the chiral kinetic theory. This means that one should add the following topological contribution to the four-current density [17, 23, 24]:

$$\delta j^\mu = \frac{e^3}{4\pi^2\hbar^2 c} \epsilon^{\mu\nu\rho\lambda} A_\nu^5 F_{\rho\lambda}, \quad (9)$$

where $A_\nu^5 = b_\nu + \tilde{A}_\nu^5$ is the axial vector potential, which is an observable quantity. Indeed, in Weyl materials, b_0 and \mathbf{b} correspond to the energy and momentum-space separation between the Weyl nodes. On the other hand, \tilde{A}_ν^5 is expressed through the deformation tensor and describes strain-induced axial (pseudoelectromagnetic) fields. In components, Eq. (9) takes the following form:

$$\delta \rho = \frac{e^3}{2\pi^2\hbar^2 c^2} (\mathbf{A}^5 \cdot \mathbf{B}), \quad (10)$$

$$\delta \mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c} A_0^5 \mathbf{B} - \frac{e^3}{2\pi^2\hbar^2 c} (\mathbf{A}^5 \times \mathbf{E}). \quad (11)$$

For \mathbf{B}_5 to be nonzero, the axial field $\tilde{\mathbf{A}}^5$ should depend on coordinates. We will assume, however, that such a dependence is weak, i.e., $\tilde{\mathbf{A}}^5$ in Eqs. (10) and (11) is negligible compared to the chiral shift \mathbf{b} .

As is easy to check, the consistent current $J^\mu = (c\rho + c\delta\rho, \mathbf{j} + \delta\mathbf{j})$ is nonanomalous, $\partial_\mu J^\mu = 0$, therefore, the electric charge is locally conserved. Note that the consistent current plays an important role even in the absence of strain-induced \mathbf{E}_5 and \mathbf{B}_5 fields. For example, in the equilibrium state with $\mu_5 = -eb_0$, the first term in Eq. (11) exactly cancels the corresponding CME current in \mathbf{j} as argued in Ref. [24]. This finding agrees with the analysis based on the band theory in crystalline solids [25]. In addition, the second term in $\delta\mathbf{j}$ correctly captures the anomalous Hall effect in the Weyl materials [26], that otherwise would be missing in the chiral kinetic theory.

Collective excitations.— By making use of the consistent current, we study the spectrum of collective excitations in the Weyl material in a constant background field $\mathbf{B}_{0,\lambda} \equiv \mathbf{B}_0 + \lambda\mathbf{B}_{0,5}$. For the sake of simplicity, we assume that the strain-induced pseudomagnetic field $\mathbf{B}_{0,5}$ is parallel to the magnetic field \mathbf{B}_0 . (We choose \mathbf{B}_0 to point in the $+z$ direction.) In addition to the external fields \mathbf{B}_0 and $\mathbf{B}_{0,5}$, collective modes could also drive dynamical deformations of the Weyl material and, therefore, induce oscillating pseudoelectromagnetic fields \mathbf{E}'_5 and \mathbf{B}'_5 . However, the corresponding fields should be extremely weak and will be neglected.

Our analysis of the electromagnetic collective modes follows the standard approach of physical kinetics [15], but generalized to the case of chiral fermions with a nonzero Berry curvature. As usual, the solution is sought in the form of plain waves, i.e., $\mathbf{E}' = \mathbf{E}'e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$ and $\mathbf{B}' = \mathbf{B}'e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$, where ω is the frequency and \mathbf{k} is the wave vector. The matter effects are captured by the polarization vector

$$\mathbf{P}^m = \chi^{mn} \mathbf{E}'^n = i \frac{\delta J^m}{\omega}, \quad (12)$$

where χ^{mn} is the electric susceptibility tensor and $m, n = 1, 2, 3$ are the spatial indices. The dispersion relations of collective modes follow from the characteristic equation for the in-medium Maxwell's equations [15]:

$$\det[(\omega^2 - c^2 k^2) \delta^{mn} + c^2 k^m k^n + 4\pi\omega^2 \chi^{mn}] = 0. \quad (13)$$

In this Letter, for simplicity, we will discuss the properties of the collective modes only in the limit $\mathbf{k} = 0$. The general case with $\mathbf{k} \neq 0$ will be reported elsewhere. In order to determine the electric susceptibility tensor, we use the consistent chiral kinetic theory, which includes the contribution to the electric current due to the Bardeen-Zumino polynomial given by Eq. (11). The distribution function is taken in the form $f_\lambda = f_\lambda^{(\text{eq})} + \delta f_\lambda$, where $f_\lambda^{(\text{eq})}$ is the equilibrium distribution function (4)

and $\delta f_\lambda = f_\lambda^{(1)} e^{-i\omega t}$ is a perturbation. To leading linear order in oscillating fields, the solution to the kinetic equation (1) reads

$$f_\lambda^{(1)} \simeq -i \frac{ev_F}{p\omega} \frac{\partial f_\lambda^{(\text{eq})}}{\partial \epsilon_{\mathbf{p}}} \left\{ (\mathbf{p} \cdot \mathbf{E}) \left[1 + \frac{\lambda \hbar e (\mathbf{B}_{0,\lambda} \cdot \mathbf{p})}{2cp^3} \right] - i (\mathbf{p} \cdot [\mathbf{B}_{0,\lambda} \times \mathbf{E}]) \frac{ev_F}{cp\omega} \right\}. \quad (14)$$

Similarly to the situation in a magnetized nonrelativistic plasma [15], the leading-order perturbation $f_\lambda^{(1)}$ is proportional to the magnitude of the oscillating electric field. By making use of this solution, we derive the following result for the polarization vector:

$$\mathbf{P} = \frac{a_0}{4\pi} \mathbf{E}' + \frac{a_1}{4\pi} (\mathbf{b} \times \mathbf{E}') + \frac{a_2}{4\pi} (\mathbf{E}' \times \hat{\mathbf{z}}), \quad (15)$$

where $\hat{\mathbf{z}}$ is the unit vector in the $+z$ direction and

$$a_0 = -\frac{\Omega_e^2}{\omega^2}, \quad a_1 = -i \frac{2e\alpha v_F}{\pi c \omega \hbar}, \quad (16)$$

$$a_2 = -i \frac{2e\alpha v_F^2}{3\pi\omega c} \sum_{\lambda=\pm} \left[\frac{B_{0,\lambda}\mu_\lambda}{\hbar^2\omega^2} - \frac{B_{0,\lambda}}{4T} F\left(\frac{\mu_\lambda}{T}\right) \right]. \quad (17)$$

Here we introduced the shorthand notations for the fine structure constant $\alpha = e^2/(\hbar v_F)$, the Langmuir (plasma) frequency

$$\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi\hbar^2} \left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3} \right)}, \quad (18)$$

and the following function of $\nu_\lambda \equiv \mu_\lambda/T$:

$$F(\nu_\lambda) \equiv -T \int \frac{dp}{p} \left\{ \frac{\partial f_\lambda^{(\text{eq})}}{\partial \epsilon_{\mathbf{p}}} - \frac{\partial \bar{f}_\lambda^{(\text{eq})}}{\partial \epsilon_{\mathbf{p}}} \right\}. \quad (19)$$

Note that the high- and low-temperature asymptotes of this function are given by $F(\nu_\lambda) \simeq 7\zeta(3)\nu_\lambda/(2\pi^2)$ for $T \rightarrow \infty$ and $F(\nu_\lambda) \simeq 1/\nu_\lambda$ for $T \rightarrow 0$, respectively. Here $\zeta(x)$ is the Riemann zeta function. In the general case, $F(\nu_\lambda)$ can be easily computed numerically.

The first term in Eq. (15) describes the high-frequency version of the Ohm's law. The second term in Eq. (15) comes from the part of the topological current Eq. (11) responsible for the anomalous Hall effect in the Weyl materials [26]. The last term in Eq. (15) proportional to the background magnetic and pseudomagnetic fields describes the usual Faraday rotation, as well as its anomalous counterpart.

By making use of Eqs. (13) and (15), we obtain the spectral equation for the collective modes

$$(1 + a_0) \left\{ 1 + \left[2a_0 + a_0^2 + a_1^2 b_\perp^2 + (a_2 - a_1 b_\parallel)^2 \right] \right\} = 0, \quad (20)$$

where we introduced the transverse $b_\perp = \sqrt{b_x^2 + b_y^2}$ and longitudinal $b_\parallel = b_z$ components of the chiral shift. Notice that the spectral equation is explicitly factorized. The corresponding solutions are

$$\omega_l = \Omega_e, \quad \omega_{\text{tr}}^\pm = \Omega_e \sqrt{1 \pm \frac{\delta\Omega_e}{\Omega_e}}, \quad (21)$$

where

$$\delta\Omega_e = \frac{2e\alpha v_F}{3\pi c\hbar^2} \left\{ 9\hbar^2 b_\perp^2 + \left[\frac{2v_F}{\Omega_e^2} (B_0\mu + B_{0,5}\mu_5) - 3\hbar b_\parallel - \frac{v_F\hbar^2}{4T} \sum_{\lambda=\pm} B_{0,\lambda} F\left(\frac{\mu_\lambda}{T}\right) \right]^2 \right\}^{1/2}. \quad (22)$$

In the absence of the chiral shift, the collective modes (21) correspond to the longitudinal ($\mathbf{E}' \parallel \hat{\mathbf{z}}$) and transverse ($\mathbf{E}' \perp \hat{\mathbf{z}}$) waves. Moreover, Eq. (21) means that the effects of dynamical electromagnetism transform, as argued in Ref. [16], the CMW into a longitudinal plasmon whose frequency coincides exactly with the Langmuir one at linear order in the (pseudo-)magnetic field. It is interesting to point out that the combined effect of the pseudomagnetic field $B_{0,5}$ and the chiral chemical potential μ_5 on the collective modes is similar to that of the magnetic field B_0 and the electric chemical potential μ . The qualitative dependence of the plasma frequencies (21) on the magnetic field B_0 is presented graphically in Fig. 1 at fixed values of b_\perp and b_\parallel .

According to the upper panel in Fig. 1, the plasma frequencies of all three collective modes are different when $b_\perp \neq 0$. In this case, the smallest splitting occurs at $B_0 = 0$, where $\omega_{\text{tr}}^+ - \omega_{\text{tr}}^- \approx \delta\Omega_e = 2e\alpha v_F b_\perp / (\pi c\hbar)$.

The situation is quite different in the case when $b_\perp = 0$, but $b_\parallel \neq 0$. This is demonstrated in the lower panel of Fig. 1. Now, while the three plasmons have generically different frequencies, one can make them degenerate by tuning the value of the magnetic field. The corresponding value of the magnetic field B_0^* , at which the frequency splitting vanishes, is given by

$$B_0^* = - \frac{4T [2v_F B_{0,5}\mu_5 - 3\hbar\Omega_e^2 b_\parallel]}{v_F [8T\mu - \hbar^2\Omega_e^2 \sum_{\lambda=\pm} F(\frac{\mu_\lambda}{T})]} - \frac{B_{0,5}\hbar^2\Omega_e^2 \sum_{\lambda=\pm} \lambda F(\frac{\mu_\lambda}{T})}{8T\mu - \hbar^2\Omega_e^2 \sum_{\lambda=\pm} F(\frac{\mu_\lambda}{T})}. \quad (23)$$

We would like to point out that these qualitative characteristics of the magnetic field dependence of the plasma frequencies should allow for a relatively simple experimental protocol to measure the values of the chiral shift in Weyl materials.

Chiral magnetic plasmons.— It is interesting to discuss the chiral features of the collective excitations in more detail. It appears that these modes, including the

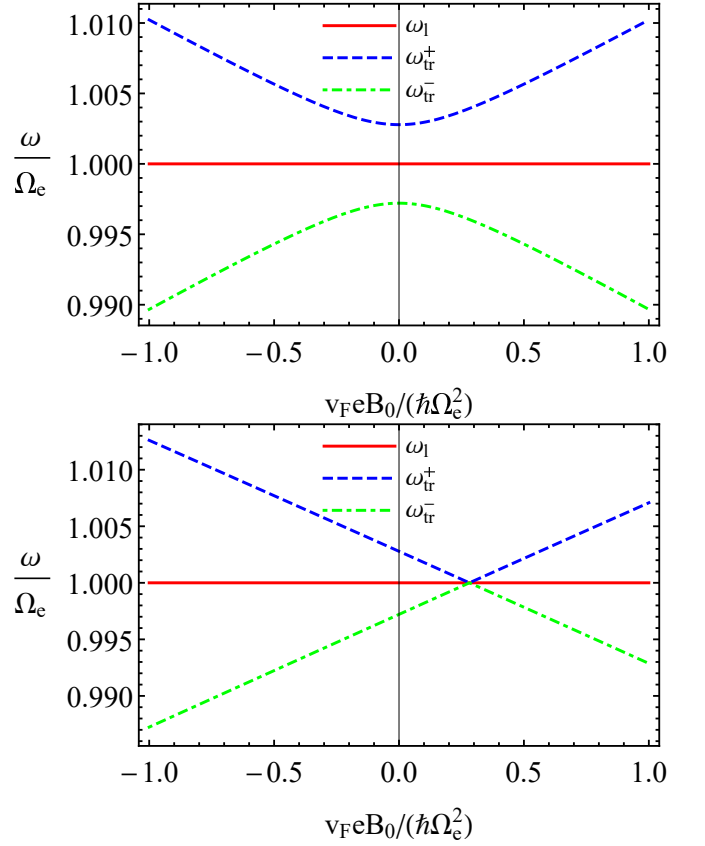


FIG. 1: (Color online) The dispersion relations of collective modes given by Eq. (21) at fixed $b_\perp = 0.2\hbar\Omega_e/e$ (upper panel) and $b_\parallel = 0.2\hbar\Omega_e/e$ (lower panel). The electric chemical potential equals $\mu = \hbar\Omega_e\sqrt{3\pi/(4\alpha)}$ and the temperature is zero.

longitudinal one, which describes the CMW with the effects of dynamical electromagnetism taken into account, are chiral plasmons, or rather chiral magnetic plasmons when a background magnetic field is present. Their chiral nature is evident from the fact that they are accompanied by oscillations of not only the electric, but also the chiral current density. The result for the oscillating part of the electric current density is clear from the polarization vector if one uses Eqs. (12) and (15). As for the oscillating part of the chiral current density, it is given by the following expression:

$$\delta\mathbf{J}_5 = \sin(\omega t)\mathbf{E} \frac{2\alpha\mu\mu_5}{3\pi^2\hbar^2\omega} - \cos(\omega t)(\mathbf{E} \times \hat{\mathbf{z}}) \frac{e\alpha v_F^2}{6\pi^2 c} \times \sum_{\lambda=\pm} \left[\frac{\lambda B_{0,\lambda}\mu_\lambda}{\hbar^2\omega^2} - \frac{\lambda B_{0,\lambda}}{4T} F\left(\frac{\mu_\lambda}{T}\right) \right]. \quad (24)$$

It is important to emphasize the topological origin of the first term in Eq. (24), which does not depend on temperature. In essence, it comes from a dynamical version of the chiral electric separation effect [27]. The second term in Eq. (24) is related to a generalized Lorentz force.

Conclusion.— As we showed in this Letter, the consistent chiral kinetic theory in Weyl materials should necessarily include the topological Chern–Simons contribution that ensures the local conservation of the electric charge in electromagnetic and strain-induced pseudoelectromagnetic fields. Moreover, as we emphasized, such a term plays an important role even in the absence of pseudoelectromagnetic fields. It allows one to correctly describe the anomalous Hall effect in Weyl materials [26] and to reproduce the vanishing CME current in an equilibrium state of chiral plasma [24, 25]. Furthermore, the topological term also affects the spectra of collective modes.

As demonstrated here, the collective modes in Weyl materials are the chiral plasmons with interesting properties. Such modes are associated with the oscillations of both electric and chiral current densities. This is in contrast to the ordinary electromagnetic plasmons which are not connected to the oscillations of the chiral current density. It is worth mentioning that for the longitudinal mode, which corresponds to the CMW, these oscillations are of purely topological origin and are related to a dynamical version of the chiral electric separation effect.

While the plasma frequency of the longitudinal mode coincides with the Langmuir one, the frequencies of the transverse modes generically split up. The frequency splitting depends on both magnetic (pseudomagnetic) field and electric (chiral) chemical potential. As we showed, the qualitative features of this dependence on the magnetic field can be used to develop a protocol for experimentally extracting both the direction and magnitude of the chiral shift parameter in Weyl materials.

It should be noted that, in this Letter, we restricted ourselves to the long-wavelength limit ($\mathbf{k} = 0$) of the chiral magnetic plasmons and used an expansion to the linear order in background magnetic/pseudomagnetic fields. The generalization of this study to the case of nonzero wave vectors ($\mathbf{k} \neq 0$) and higher orders in magnetic/pseudomagnetic fields will be reported elsewhere.

The work of E.V.G. was supported partially by the Program of Fundamental Research of the Physics and Astronomy Division of the NAS of Ukraine. The work of V.A.M. and P.O.S. was supported by the Natural Sciences and Engineering Research Council of Canada. The work of I.A.S. was supported by the U.S. National Science Foundation under Grant No. PHY-1404232.

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